

SET THEORY HOMEWORK 2

Due Monday, February 20.

Problem 1. In ZF^- prove the Schröder-Bernstein theorem i.e. that if $A \preceq B$ and $B \preceq A$ implies that $A \approx B$.

Hint: Suppose $f : A \rightarrow B$ and $g : B \rightarrow A$ are one-to-one. Set $A_0 = A$, $B_0 = B$, $A_{n+1} = g''B_n$, $B_{n+1} = f''A_n$, $A_\infty = \bigcap_n A_n$, $B_\infty = \bigcap_n B_n$. Let $h(x)$ be $f(x)$ if $x \in A_\infty \cup \bigcup_n (A_{2n} \setminus A_{2n+1})$. Otherwise let $h(x)$ be $g^{-1}(x)$. Show that h is well defined and $h : A \rightarrow B$ is one-to-one and onto.

Problem 2. Assume CH (but not GCH). Show that for every natural number $n > 0$, $\omega_n^\omega = \omega_n$.

Problem 3. (1) Show that if $\aleph_\omega < 2^\omega$, then $(\aleph_\omega)^\omega = 2^\omega$.
(2) Show that $\aleph_\omega^{\omega_1} = 2^{\omega_1} \cdot \aleph_\omega^\omega$.

Problem 4. Suppose that κ is inaccessible. Show that $|V_\kappa| = \kappa$ and $V_\kappa \cap ON = \kappa$.